

History of Logic from Aristotle to Gödel

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Stoic Logic: The Dialectic from Zeno to Chrysippus

THE STOIC PHILOSOPHY OF LANGUAGE

"The nature of the Stoics' philosophy of language is the most tantalizing problem in the history of semantics. We know enough of it to say that it was by far the most intricate and probably the most insightful theory of its kind in antiquity and for centuries afterward; but we cannot be certain what its details were, and even its leading principles are sometimes obscured by vague or conflicting testimony. Those Stoics who had most to say about language were, naturally, the logicians, and the difficulty of determining the exact character of what they had to say stems from the fact that none of the many works of the Stoic logicians is extant. The best surviving sources (which date from almost five hundred years after the period of greatest development in Stoic logic and semantics) are Sextus Empiricus, *Outlines of Pyrrhonism*, Book II, and *Adversus Mathematicos*, Book VIII; and Diogenes Laërtius, Book VII. Under these circumstances it is seldom possible to assign a particular doctrine to a particular Stoic, but much of the best of their logic and semantics is very likely to be the work of Chrysippus (c. 280–206 BCE). Under the Stoic division of philosophy into physics, ethics, and logic, logic was divided into rhetoric and dialectic, and dialectic further divided into an account of language (*περί της φωνης*) and an account of things signified (*περί των σημαινόμενων*). Both these subdivisions contain material relevant to semantics. In their account of language the Stoics distinguished vocal sound generally, "which may include mere noise," from the sort that is articulate (*ἔξαρθρος*), that is, capable of being embodied in written symbols (*ἐγγάμματος*). Articulate sound, in turn, may be non-significant—for instance, "blityri"—or significant (*σημαντηρή*); but for any articulate sound to be considered a sentence (*λόγος*) it must be significant and a product of someone's reason (Diogenes Laërtius 7.55–57).

Within that same branch of their dialectic the Stoics recognized five kinds of words and distinguished their semantic or syntactic functions. They were the first who clearly separated (1) names, such as "Socrates," from (2) appellatives (*προσηγορίαί*), such as "man." (Cf. Aristotle's similar but significantly different distinction in *De Interpretatione*, Ch. 7.) A name "*points out* a kind proper to an individual," while an appellative "*signifies* a common kind." (3) A verb "*signifies* a predicate"; (4) a conjunction "*binds together* the parts of a sentence"; (5) an article (possibly also what would now be called a relative pronoun) serves to "*distinguish* the gender and number of nouns" (Diogenes Laërtius 7.58). Thus the function of conjunctions and articles is purely syntactic, the semantic function of (proper) names is different from that of appellatives (or common names), and the appellative and the verb—the standard ingredients of the simplest kind of logicians' sentence—have one and the same kind of semantic function. The appellative occurring in a sentence signifies a subject and the verb a predicate or "something attachable (*συστακτόν*) to the one or more subjects."

Obviously the division between the accounts of language and of things signified was not exclusive, but the transition from the one account to the other as the Stoics conceived of them may be seen in the claim that all we utter (*προφέρειν*) is sounds, while what we express (*λέγειν*) is matters of discourse (*πράγματα*), or *lekta*—"expressibles" (Diogenes Laërtius 7.57). It is the doctrine of the *lekton* around which the Stoics organized their account of things signified. In its novelty, importance, and difficulty that doctrine overshadows all the considerable remainder of their philosophy of language." (p. 757).

From: Norman Kretzmann, *Semantics, History of* in: *Encyclopedia of Philosophy. Second edition*. Edited by Borchert Donald M. New York: Thomson Gale 2006. pp. 750-807.

THE REDISCOVERY OF STOIC LOGIC

"The first reactions to the negative appraisal of Stoic philosophy have come not from historians or philosophers specializing in antiquity, but from logicians being interested in the development of ancient logic.

(...)

Now in addition to what has been said in connection with the nineteenth-century misinterpretations and misconceptions, let me quote another view about the specific reasons for the disappreciation as well as for the rehabilitation of Stoic logic; it is found in I. M. Bochenski's *Ancient Formal Logic* (Amsterdam, 1951), and it clearly portrays the difference in attitude of the logicians of the twentieth century towards the Stoic logical system:

Modern history of Logic had been started during the XIXth century, but its state was very bad at that time -- indeed until 1930 approximately -- because of two phenomena. On one hand, most of the historians of logic took for granted what Kant said on it; namely that 'formal logic was not able to advance a single step (since Aristotle) and is thus to all appearance a closed and complete body of doctrine'; consequently, there was, according to them, no history of logic at all, or at the most, a history of the decay of Aristotelian doctrines. On the other hand, authors writing during that period were not formal logicians and by 'logic' they mostly understood methodology, epistemology and ontology. . . . We may place the beginning of recent research in our domain in 1896 (when Peirce made the discovery that the Megarians had the truth-value definition of implication. (pp. 4-5)

Now whether it is Peirce to whom we owe the revival of interest in Stoic logic or not, what certainly is the case is that, from the early decades of the twentieth century on, given the important developments in the field of symbolic logic, it has finally become obvious that Stoic logic differed essentially from Aristotelian logic and should be studied on its own merits. The articles and books on Stoic logic which since then have been published, have examined in detail the Stoic contribution to the development of a logical calculus:

- J. Lukasiewicz, 'Zur Geschichte der Aussagenlogik', *Erkenntnis*, 5 (1935).
- B. Mates, *Stoic Logic* (Berkeley, 1953).
- O. Becker, *Zwei Untersuchungen zur antiken Logik* (Wiesbaden, 1957).
- W. and M. Kneale, *The Development of Logic* (Oxford, 1962).
- M. Mignucci, *Il significato della logica stoica* (Bologna, 19672).
- I. Mueller, 'Stoic and Peripatetic Logic', *Archiv fur Geschichte der Philosophie*, 51 (1969), 173-87.
- M. Frede, *Die stoische Logik* (Gottingen, 1974).
- M. Frede, 'Stoic vs. Aristotelian syllogistic', *Archiv fur Geschichte der Philosophie*, 56 (1974), 1-32."

From: Katerina Ierodoakonou, Introduction. The Study of Stoicism: Its Decline and its Revival, in: K. Ierodiakonou (ed.), *Topics in Stoic Philosophy* - Oxford, Clarendon Press, 1999 pp. 15-17

"Modern mathematical logic has taught us to distinguish within formal logic two basic disciplines, no less different from one another than arithmetic and geometry. These are, the logic of propositions and the logic of terms. The difference between the two consists in the fact that in the logic of propositions there appear, besides logical constants, only propositional variables, while in the logic of terms term variables occur.

The simplest way of making this difference clear is to examine the Stoic and the Peripatetic versions of the law of identity. To avoid misunderstanding let me at once say that, so far as our sources indicate, the two laws of identity were only incidentally formulated by the ancients, and in no way belong to the basic principles of either logic. The Stoic law of identity reads "if the first, then the first", and is to be found as a premiss in one of the inference-schemata cited by Sextus Empiricus. (1) The Peripatetic law of identity is "a belongs to all a", and is not mentioned by Aristotle, but can be inferred from a passage in Alexander's commentary on the *Prior Analytics*. (2) Using variable letters we can write the Stoic law of identity in the form "if p then p"; the Peripatetic law can be recast in the form "all a is a". In the first law the expression "if ... then" is a logical constant, and "p" a propositional variable; only propositions such as "it is day" can be meaningfully substituted for "p". This substitution yields a special case of the Stoic law of identity: "if it is day, it is day". In the second law the expression "all ... is" is a logical constant, and "a" a term variable; "a"

can be meaningfully replaced only by a term, and, in accordance with a tacit assumption of Aristotelian logic, only by a general term at that, such as "man" Upon substitution we get a special case of the Peripatetic law of identity: "all man is man". The Stoic law of identity is a thesis of the logic of propositions, whereas the Peripatetic law is a thesis of the logic of terms.

This fundamental difference between the logic of propositions and the logic of terms was unknown to any of the older historians of logic. It explains why there has been, up to the present day, no history of the logic of propositions, and, consequently, no correct picture of the history of formal logic as a whole. Indispensable as Prantl's 3) work is, even today, as a collection of sources and material, it has scarcely any value as an historical presentation of logical problems and theories. The history of logic must be written anew, and by an historian who has fully mastered mathematical logic. I shall in this short paper touch upon only three main points in the history of propositional logic. Firstly I wish to show that the Stoic dialectic, in contrast to the Aristotelian syllogistic, is the ancient form of propositional logic; and, accordingly, that the hitherto wholly misunderstood and wrongly judged accomplishments of the Stoics should be restored their due honour. Secondly I shall try to show, by means of several examples, that the Stoic propositional logic lived on and was further developed in medieval times, particularly in the theory of "consequences". Thirdly I think it important to establish something that does not seem to be commonly known even in Germany, namely that the founder of modern propositional logic is Gottlob Frege.

(...)

The fundamental difference between Stoic and Aristotelian logic does not lie in the fact that hypothetical and disjunctive propositions occur in Stoic dialectic, while in Aristotelian syllogistic only categorical propositions appear. Strictly speaking, hypothetical propositions can be found in Aristotle's syllogistic also, for each proper Aristotelian syllogism is an implication, and hence a hypothetical proposition. For example, "If a belongs to all b and c belongs to all a, then c belongs to all b". (7) The main difference between the two ancient systems of logic lies rather in the fact that in the Stoic syllogisms the variables are propositional variables, while in Aristotle's they are term variables. This crucial difference is completely obliterated, however, if we translate the above-mentioned Stoic syllogism as Prantl does (I, p. 473):

If the first is, the second is

But the first is

Therefore the second is.

By adding to each variable the little word "is", which occurs nowhere in the ancient texts, Prantl, without knowing or wishing it, falsely converts Stoic propositional logic into a logic of terms. For in Prantl's schema only terms, not propositions, can be meaningfully substituted for "the first" and "the second". As far as we can judge from the fragmentary state of the Stoic dialectic that has come down to us, all Stoic inference-schemata contain, besides logical constants, only propositional variables. Stoic logic is therefore a logic of propositions. (8)" pp. 197-200

(1) Sextus, *Adv. Math.* VIII 292 (missing in Arnim): *ei to poton, to poton*. Good as H. von Arnim's collection is (*Stoicorum veterum fragmenta* [SVF], vol. II, Leipzig 1903), it does not begin to serve as source material for Stoic dialectic.

(2) Alexander, *In anal. pr. comm.*, ed. Wallies, p. 34, l. 19.

(7) Aristotle, *An. pr. II.* 11. 61b34

(8) I have defended this interpretation of the Stoic dialectic since 1923; see J. Lukasiewicz, "Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagen-kalküls", *Comptes rendus des séances de la Société des Sciences et des Lettres de Varsovie* 23 (1930), cl. III, pp. 51-77. ["Philosophical Remarks on Many-Valued Systems of Propositional. Logic", pp. 153-178 of this volume.] I rejoice in having found in H. Scholz, *Geschichte der Logik* (Berlin, 1931), p. 31, a supporter of this point of view.

From: Jan Lukasiewicz - *On the history of the logic of proposition* (1934) - Translated in: *Selected Works* - Edited by Ludwik Borkowski - Amsterdam, North-Holland, 1970 pp. 197-217 (Greek omitted)

"I have compiled thus many quotations on purpose, for, although they illuminate one of the most important problems of logic, it nevertheless appears that many of them were either unknown to the historians of logic, or at least not sufficiently appreciated. The reason for this is in my opinion that the history of logic has thus far been treated by philosophers with insufficient training in logic. The older authors cannot be blamed for this, as a scientific logic has existed only for a few decades. The history of logic must be written anew, and by an historian who has a thorough command of modern mathematical logic. Valuable as Prantl's work is as a compilation of sources and materials, from a logical point of view it is practically worthless. To give only one illustration of this, Prantl, as well as all the later authors who have written about the logic of the Stoa, such as Zeller and Brochard, have entirely misunderstood this logic. For anybody familiar with mathematical logic it is self-evident that the Stoic dialectic is the ancient form of

modern propositional logic. (26)

Propositional logic, which contains only propositional variables, is as distinct from the Aristotelian syllogistic, which operates only with name variables, as arithmetic is from geometry. The Stoic dialectic is not a development or supplementation of Aristotelian logic, but an achievement of equal rank with that of Aristotle. In view of this it seems only fair to demand of an historian of logic that he know something about logic. Nowadays it does not suffice to be merely a philosopher in order to voice one's opinion on logic.

(26) I have already expressed this idea, in 1923, in a paper read to the first congress of Polish philosophers in Lwow. A short summary of it appeared in *Przegląd Filozoficzny* 30 (1927), p. 278. [Lukasiewicz develops his historical analysis of Stoic logic in his article "On the History of the Logic of Propositions" (pp. 197-217 of this book).] p. 178

From: Jan Lukasiewicz, *Philosophical Remarks on Many-Valued Systems of Propositional Calculus* (1930) - Translated in: *Selected Works* - Edited by Ludwik Borkowski, Amsterdam, North-Holland, 1970 pp. 153-178

"In the first comprehensive history of western logic Prantl (1) described Stoic logic as "dull," "trivial," and "pedantic." Prantl's dismissal of Stoic logic was accepted by most interpreters of Stoicism for three quarters of a century. However, since the publication of Lukasiewicz's article, "On the History of the Logic of Propositions" in 1934, (2) Prantl's evaluation has been largely abandoned. Bochenski's remark, "The development of formal logic in antiquity reached its peak in the works of the thinkers belonging to the Megaric and Stoic Schools," exemplifies well the radical rehabilitation of the Stoics as logicians. (3) The cause of this rehabilitation is not the discovery of new texts, but rather the twentieth-century revolution in the subject of logic itself. Lukasiewicz and others, working with a full understanding of modern logic, have succeeded in retrieving from the ancient texts a Stoic logical theory of startling originality which rivals the achievement of Aristotle, the founder of logic. The failure of Prantl and his successors to accomplish this retrieval stems not from their obtuseness or stupidity but from the fact that the background scientific knowledge needed to understand the Stoic achievement was not available to them.

A factor contributing to Prantl's low opinion of Stoic logic was the character of the ancient texts themselves. There are no primary sources for Stoic logic analogous to Aristotle's *Prior Analytics*, and the ancient secondary sources are brief and usually hostile in their treatment of the subject. In many cases Prantl's evaluations simply repeat or develop remarks in the sources themselves. The unsatisfactoriness of the sources (on this see Mates, *Stoic Logic* 8-10) makes any but a tentative reconstruction of Stoic logic impossible. Unless an indication is given to the contrary, what I describe will be the most certain features of the theory.

One of the uncertain features is chronology. The history of Stoicism proper covers five centuries during which the logical theory, like other doctrines of the school, underwent modification and development. In the case of logic we know of some disagreements within the school and some ideas that can be ascribed to individuals, but most of our sources refer simply to "the Stoics," as if there were a single, unambiguous Stoic logical theory. Commentators have tended to assign the major Stoic achievements in logic to Chrysippus (c. 280 B.C. - c. 206 B.C.), the third leader of the Stoa, of whom it was said, "If there were a dialectic among the gods, it would be none other than the Chrysippean one." (Diogenes Laertius 7.180. At 7.198 Diogenes mentions that Chrysippus wrote 311 books on logical matters.) In general I shall not attempt to assign logical doctrines to specific persons, but simply speak of "Stoic logic." Occasionally, however, it will be necessary to refer to possible disagreements within the school." pp. 1-2

(1) C. Prantl, *Geschichte der Logik im Abendlande* (Leipzig, 1855) 408. I have generally given at most one ancient source for a doctrine. More information about sources can be found by consulting B. Mates, *Stoic Logic* (2nd ed.) or M. Frede, *Die Stoische Logik*.

(2) Reprinted in *J. Lukasiewicz, Selected Works*, ed. L. Borkowski

(3) I. M. Bochenski, *Ancient Formal Logic* (Amsterdam, 1951) 77.

From: Ian Mueller, *An Introduction to Stoic Logic*. In *The Stoics*. Edited by John M. Rist, Berkeley: University of California Press 1978. pp. 1-26

"Zeno, the founder of the Stoic school of philosophy, is said to have been influenced primarily by two of the Socratic schools, the Cynics and the Megarians. (*) From the Cynics, according to the usual account, he took his moral teaching; from the Megarians, his logic. In view of our present subject, we shall omit all discussion of the Cynics and devote our attention to the Megarians.

The Megarian school was founded by Euclid, a follower of Socrates and a somewhat older contemporary of Plato. (See fig. 1.)

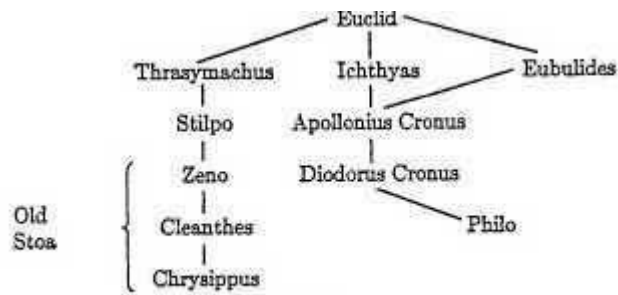


Fig. 1.

Among the pupils of Euclid were: Eubulides, a famous logician to whom the antinomy of The Liar is sometimes ascribed; Ichthyas, the successor of Euclid as head of the school; and Thrasymachus of Corinth, who is known primarily as the teacher of Stilpo. Stilpo, a contemporary of Aristotle, enjoyed a great reputation as a lecturer. He is supposed to have been somewhat influenced by the Cynics. His most famous pupil was Zeno, founder of Stoicism. Another important branch of the Megarian school consisted of Eubulides, Apollonius Cronus, Diodorus Cronus, and Philo, in that order. The latter two are very important in connection with Stoic logic, mainly for their views on the truth-conditions of conditionals.

Diodorus, a native of Iasus in Caria, lived at the court of Alexandria in the reign of Ptolemy Soter. His surname or nickname "Cronus" ("old fool") is variously explained. According to one story, it was given to him by Ptolemy on account of his inability to solve a problem of logic put forth by Stilpo at a royal banquet. In fact, Diodorus is said to have taken his defeat so much to heart that he went home, wrote a treatise on the subject, and died in despair. According to another account, Diodorus took the surname from his teacher, Apollonius Cronus. At any rate, Diodorus was certainly not regarded as an old fool in antiquity. On the contrary, he was so celebrated for his dialectical skill that he was called "the logician" and "most logical one". This epithet gradually became a surname, and was even applied to his five daughters, who were also distinguished as logicians.

Little is known of the philosophy of Diodorus save two important definitions (and examples illustrating these): (1) a proposition is possible if and only if it either is true or will be true; (2) a conditional proposition is true if and only if it neither is nor was possible for the antecedent to be true and the consequent false. It is known that he constructed the famous "Master" argument (*kurieuon logos*) to justify his definition of "possible." It is also known that he entered into a controversy with his pupil Philo over the truth-conditions for hypothetical propositions; this controversy was perpetuated and enlarged within the Stoic school. (**)

Philo of Megara, the pupil of Diodorus, was also very famous as a logician. Almost nothing is reported of his life except that he was a friend of Zeno. Chrysippus later wrote treatises against both him and his master. Philo disagreed with Diodorus concerning the nature of possibility and especially concerning the criterion for the truth of conditional propositions. Regarding the first, he thought (as against Diodorus) that a piece of wood at the bottom of the sea should be considered combustible even if it will never be burned. In regard to conditionals, he gave exactly the modern truth-table definition: a conditional is false if it has a true antecedent and a false consequent; in the other three cases it is true.

Zeno himself apparently lived ca. 350-260 B.C., but the dates are very uncertain. Like all the other major Stoic philosophers before the Christian era, he was not a native of Greece proper. (His birthplace was at Citium, in Rhodes.) Few facts are known about him, but where the facts leave off, legend begins. It is said that he was greatly respected for his personal characteristics dignity, modesty, sincerity, affability. Presumably because of a life of moderation, he lived to the ripe old age of ninety-eight, and, as the story has it, he died in the following way. As he was leaving the school one day, he stumbled and broke his toe. Beating his hand upon the ground, he addressed himself to the gods: "I'm coming of my own accord. Why then do you bother to call me?" Then he perished by holding his breath.

Also according to the legends, Zeno devoted much thought and energy to proposed reforms in language. This aroused ire in certain quarters, and it was pointed out that he was proposing to reform a language which he himself could hardly speak. As he was fond of coining new words, much of the technical vocabulary of Stoic logic may well be attributed to him. It was said that he used new terms in order to conceal his plagiarism of the views of his predecessors; Cicero repeats this charge at least fourteen times. His writings, which were not numerous and were written in a very poor style, have been lost (excepting, of course, a few fragments).

The second head of the Stoic school was Cleanthes, known throughout antiquity as a man of strong character, great energy, and weak intellect. According to one story, he was a prize fighter who came to Athens with four drachmas in his pocket and entered the school of Zeno. He accepted Zeno's teaching in every detail and passed it on unchanged. At the age of ninety-nine or so, he died by starving himself to death.

Cleanthes was succeeded by Chrysippus, often said to have been the greatest logician of ancient times. Chrysippus was regarded as the second founder of Stoicism; according to an old saying, "If there had been no Chrysippus, there would have been no Stoa." He was born in 280 B.C. in Cilicia; the date of his death may be conjectured as 205 B.C. Without doubt, he was the best student his Stoic professors ever had. While in training, he thought of so many skeptical arguments against Stoicism that he was accused by the later Stoics of supplying Carneades with ammunition for attacking them. Chrysippus wrote 750 books, if the list given by Diogenes can be trusted. Of these we possess only the titles and a small number of fragments. But the titles alone show that he wrote on almost every important aspect of propositional logic. There are many ancient complaints that Chrysippus' books were dry and repetitious, and written in a very poor style. Yet they were widely read. He did not, like Cleanthes, merely repeat the words of his predecessors; there is a story that when he was a student of logic he wrote to Cleanthes, "Just send me the theorems. I'll find the proofs for myself."

It seems likely that Chrysippus was responsible for the final organization of Stoic logic into a calculus.

When the five basic undemonstrated argument-types are cited, the name of Chrysippus is usually mentioned; in one place it is expressly stated that Chrysippus restricted the number of these types to five."

(*) For the following account I am indebted to Zeller, *Die Philosophie der Griechen*, vol. 2, part 1, pp. 244 ff., and vol. 3, part 1, pp. 27-49; William Smith, *Dictionary of Greek and Roman Biography and Mythology* (Boston, Little, Brown, 1849), 3 vols.

(**) The views of Diodorus will be discussed fully in the sequel, pp. 36-40, 44-51. Cf. my article, "Diodorean Implication."

From: Benson Mates, *Stoic Logic*, Berkeley, University of California Press 1953, pp. 5-7

SUMMARY OF BENSON MATES, *STOIC LOGIC* (1953)

Chapter I. *Introduction*: "The aim of this study is to present a true description of the logic of the Old Stoa. It repeats most of Lukasiewicz's published conclusions on the subject and offers additional evidence for them. It also (1) describes the Stoic semantical theory and compares it with certain similar modern theories, (2) attempts to give a better account of the heretofore misunderstood Diodorean implication, (3) points out the Stoic version of the conditionalization principle, and (4) discusses the contention of the Stoics that their propositional logic was complete. In appendices it offers and justifies new translations of some important fragments pertaining to Stoic logic. The Stoic authors in whose work we shall be interested primarily are Zenon, Cleanthes, and Chrysippus. Closely associated with them were Diodorus Cronus and Philo, of the Megarian school. Since the writings of these men have been lost, and since our sources usually do not distinguish between the views of the various Stoics, we are forced to treat the entire Old Stoa as a unit. This, of course, creates many difficulties. The best of our sources are Sextus Empiricus and Diocles Magnes (*apud* Diogenes Laertius). We also derive bits of information from Cicero, Gellius, Galen, Boethius, Apuleius, Alexander of Aphrodisias, Simplicius, Philoponus, Origen, Proclus, Stobaeus, Epictetus, Seneca, and a few others. Of these, only Epictetus and Seneca were favorably inclined toward Stoicism, and they, unfortunately, restricted their attention almost entirely to ethics. It is thus remarkable that the fragments of Stoic logic, transmitted by unsympathetic hands, are as clear and consistent as they are." p. 1

Chapter II. *Signs, sense, and denotation*: "The Chapter is divided into two sections. The first contains an account of the Stoic distinction between the sign, the significate (called the "Lekton"), and the physical object to which the sign refers. Various types of signs and their corresponding Lekta are described in detail. In the second section the Stoic theory is compared with the modern theories of Frege and Carnap and is shown to bear marked resemblance to them, particularly in regard to what Carnap calls the "intension" of linguistic expressions. Numerous dissimilarities are also indicated, the most important of which are: (1) the Stoics restricted the denotation of expressions to bodies; (2) the Stoics did not take truth-values as the denotations of sentences." p. 11

Chapter III. *Propositions, truth, and necessity*: "This Chapter is divided into three sections. The first defines and classifies propositions and discusses their fundamental properties. A proposition is said to be "a complete Lekton assertoric in itself." Its most basic property is that of being true or false and not both. Propositions are classified as atomic and molecular; each of these classes in turn is divided into several subclasses. The absence from Stoic logic of examples beginning with "all" is noted. In the second section, the many Stoic usages of the words "truth" and "true" are taken up *seriatim*. All these usages are definable in terms of the usage referring to propositions. The third section deals with Stoic notions of necessity and possibility, as found in the fragments of certain (Megarian) philosophers to whom the notions were

originally due. It is shown that a reference to time plays a very important role in Diodorus' view of possibility. (This is closely connected with his position in the controversy over implication, to be discussed in chap. IV.) A brief account of what is known of the famous "Master" argument of Diodorus is included, together with a few remarks on the views of Philo and Chrysippus regarding possibility." p. 27

Chapter IV. *Propositional connectives*: "The Stoics gave truth-functional definitions of all the more important propositional connectives, and defined also some non-truth-functional connectives. These definitions, and the various controversies over them, form the subject matter of the present chapter. The first section, on implication, contains an account of the four-sided argument over the truth-conditions for hypothetical propositions. It is shown that Philo's type of implication was exactly the same as the modern "material implication." Diodorean implication is defined and distinguished from Chrysippean implication, which is the ancient equivalent of what is now called "strict implication." The connection between Diodorus' views on implication and on necessity is shown. In the second section we are concerned with disjunction. The Stoics distinguished between inclusive and exclusive disjunction, gave truth-functional definitions of both types and also a non-truth-functional definition of the latter type. The third section considers conjunction, along with several other connectives. In the fourth section, we see how implication was defined in terms of conjunction and negation; also, how exclusive disjunction was defined in terms of negation and equivalence. Certain difficulties in the evidence for these definitions are pointed out." p. 42

Chapter V. *Arguments*: "This Chapter consists of five sections. In the first, "argument" is defined as "a system of propositions composed of premises and a conclusion." A valid argument, according to the Stoics, is an argument such that the negation of its conclusion is incompatible with the conjunction of its premises. A true argument is a valid argument which has true premises, and a demonstration is a special kind of true argument. Another subclass of the valid arguments contains the so-called "undemonstrated" arguments; of these, five types were called "simple" and the innumerable others were called "non-simple," or "derived." To achieve generality in their discussions of propositional logic the Stoics made use of inference-schemas containing the numerals "first," "second," and so on as propositional variables. The second section contains an exposition of the five basic undemonstrated argument-types, as they are described in some twelve sources. The third section discusses an important Stoic principle which is closely related to the so-called "deduction theorem." In the fourth section is an account of the Stoic method of deriving non-simple undemonstrated arguments from simple ones; examples are considered in detail. Note is taken of the assertion of the Stoics that their propositional logic was complete. The fifth section describes the Stoic classification of invalid arguments and also considers briefly the famous paradox of The Liar, which was the subject of much Stoic writing. The classification is found to be poor, but the Stoic version of The Liar is stronger than the usual Epimenides paradox." p. 58

Chapter VI. *Evaluations of Stoic logic*: "In this concluding chapter we consider the traditional evaluations of Stoic logic, together with some of the confusions upon which they are based. The first section concerns some typical adverse criticisms by Prantl and Zeller. Unfortunately, these cannot be challenged by attacking the relevance or accuracy of the evidence for them, since there is no evidence for them. But it is apparent that Prantl and Zeller did not understand Stoic logic. The second section discusses the great confusion which exists in regard to the meaning of the technical term *sunemmenon*. Third, there is a short conclusion." p. 86

Appendix A. *Translations*: "This Appendix consists of translations of some of the fragments which comprise our sources for Stoic logic. I have included only the fragments upon which relatively important sections of this study rest, and, of these, only passages which have not already been adequately translated into English." p. 95

Appendix B. *Glossary*: "This Glossary is not intended to be a complete list of the technical terms in Stoic logic. It includes only terms that appear in a sufficient number of contexts to establish their technical usage. Further, only a few of the more important occurrences of each term are cited. Usually these will include a definition or at least a passage of relatively clear meaning. Other glossaries of Stoic terminology are as follows:

R. G. Bury, *Sextus Empiricus*, volume 3. This glossary is almost worthless in regard to logical terminology.

I. M. Bochenski, *Elementa Logicae Graecae*, pp. 99 ff. (Greek-Latin). Good.

J. W. Stakelum, *Galen and the Logic of Propositions*, pp. 92-93 (Greek-English). Good.

See also the *Index Verborum* in volume 4 of *Stoicorum Veterum Fragmenta*. Most of the Aristotelian commentators are well indexed, but unfortunately the indices for Sextus are very incomplete, and there are none for Diogenes Laertius." 132

SOURCES FOR STOIC LOGIC

"Except for a few fragments, all the writings of the earlier Stoics have been lost. We must therefore depend on secondary sources. But that is only half of the difficulty. Since none of the later Stoics had much to say about logic, we are in the very unsatisfactory position of having to depend on the accounts of men who were without exception opponents of the Stoics. In view of this, it is all the more remarkable that Stoic logic makes as excellent a showing as it does. Perhaps the saving circumstance was that the essentials of Stoic logic were brought together in handbooks not long after the time of Chrysippus. Such handbooks were commonly entitled "Introduction to Logic" (*eisagogé dialektiké*), and evidently had a very wide circulation. Whatever accuracy and sense remain in the bits of Stoic logic which have filtered down to us probably derive from the fact that our sources made use of the handbooks.

The difficulties created by the loss of the Stoic writings are even greater than might at first appear. Since our sources do not distinguish between the views of the various Stoics but rather tend to ascribe the sayings of any of them to all of them, we must treat the school as a whole, even though we know that this procedure will lead to apparent inconsistencies. Also, it is obvious that technical writings such as those on logic suffer from being reported at second hand; of all our sources, Sextus is the only one who seems to have had some understanding of the theory he was reporting. Another serious difficulty arises from the fact that our best sources are at least four hundred years later than Chrysippus. By this time the mixture and confusion of Stoic logic with that of Aristotle were well under way, producing strange conglomerates like that found in Galen's *Institutio Logica*. Since we do not possess the information necessary for disentangling the two doctrines, we can only make the best of it.

Far and away our most important source for Stoic logic is Sextus Empiricus, a Greek physician and Skeptic, who lived in the first half of the third century of the Christian era. Almost nothing is known of his life. Two of his works are extant, the *Outlines of Pyrrhonism*, in three books, and *Against the Mathematicians*, in eleven books. Most of his discussion of Stoic logic is to be found in Book II of the *Outlines* and Book VIII of *Against the Mathematicians*; the accounts given in these two places are often identical. Sextus is our only intelligent source. But even with him there is a fly in the ointment: he quotes the Stoics only to refute them. We may expect, therefore, that any parts of Stoic logic which he found either too difficult or too good to refute will be absent from his account. Also, he emphasized those matters on which Stoic opinions differed, with the result that we get no clear statement of the logical doctrine of any one man.

The next best picture of Stoic logic is that given by Diogenes Laertius, author of *Lives of Eminent Philosophers*. There is no information whatever on his own life, but since Sextus and Saturninus are the latest writers he quotes, it is sometimes guessed that he lived in the third century of the Christian era. As is well known, Diogenes is wholly unreliable on many subjects. It is therefore fortunate for us that in writing his life of Zeno (Book VII) Diogenes had recourse to a book written by Diodes Magnes, a scholar of the first century B.C., who seems to have had a fair knowledge of Stoic logic. The most serious deficiency of Diogenes' account is its extreme brevity; what there is of it is as excellent as anything to be found in Sextus. All our other sources for Stoic logic are relatively unsatisfactory. Scattered references to the Stoa will be found throughout the twenty volumes of Galen's works,(8) but discussions of any extent are rare. The little treatise called *Historia Philosopha* contains the remains of a good account of the five basic undemonstrated argument-types. However, it has been necessary for editors to reconstruct the text on the analogy of corresponding passages in Sextus; consequently it has little independent value. There is also the handbook, *Institutio Logica*, ascribed to Galen by the manuscripts. Prantl has vehemently challenged its authenticity; Kalbfleisch has "proved" it genuine with equal vigor.(9) In any case, the treatise is of considerable interest to historians of logic. Although it is a mixture of Aristotelian and Stoic logic, its account of the five basic types of argument is clear and agrees exactly with our other information. Its criticism of these, however, is typically Peripatetic and typically confused. The treatise contains a few further hints about the views of the Stoics, but nothing else of value for our purpose.

Other scraps of information are to be found in the writings of Cicero, Gellius, and the many Aristotelian commentators.(10) Most of these scraps fit consistently into the picture, but they are too brief to be of much help.

The work of the later Aristotelian commentators reveals extreme confusion between Stoic and Aristotelian logic, and hence is of very little use as a source.

All our sources have one characteristic in common: the more interesting the logic becomes, the more corrupt the text becomes. Because of the technical terminology and the very unusual sentences with which the Stoics sometimes illustrated their points, the origin of these textual difficulties is understandable -- but the difficulties remain. Especially is this noticeable in Galen's *Institutio Logica*, where occasionally the whole thread of argument is lost.

In view of all these difficulties, the reader may well wonder whether there is enough evidence to justify the attempt to give a complete account of Stoic logic. He may answer this question for himself by reading the

following chapters and, if he is interested, by checking the exposition against the Stoic passages which are cited. He will find that no effort has been made to conceal or minimize evidence contrary to the various theses proposed; the price exacted by this procedure is that the account is not always as simple and clear as one might desire."

(8) The best exegetical study of the logic of Galen is by Stakelum, *Galen and the Logic of Propositions* [1940]. See especially the summary, pp. 90-91. [cited below]

(9) Prantl, *Geschichte der Logik im Abendlande*, [1855] pp. 591-610; Kalbfleisch, "Ueber Galens Einleitung in die Logik," [1897] pp. 681-708.

(10) The relevant writings of these authors are listed in the Bibliography. An excellent critical discussion of Apuleius, Alexander of Aphrodisias, Sextus, Diogenes Laertius, Themistius, Boethius, Ammonius, Simplicius, and Philoponus as sources of information about ancient logic may be found in Bochenski, *La Logique de Théophraste*, [1947] chap. I.

From: Benson Mates, *Stoic Logic*, Berkeley, University of California Press 1953, pp. 8/10

"Conclusion.

It is evident from the *Introduction to Dialectic* that Galen was perfectly familiar with Stoic formal logic. It is equally evident that he was not a Stoic. Because he began his studies in the Stoic school, his terminology was Stoic, and the powerful Stoic influence of his period confirmed him in Stoic usage. His strong formalistic tendency also manifests his Stoic training. His later profession of Aristotelianism, however, becomes certain in his Peripatetic interpretation of Stoic propositions.

Galen presents exactly the forms of Stoic compound propositions, which he calls "hypothetical"; but his explanation are not Stoical, because he follows the Peripatetic practice of examining the matter of the propositions; instead of the Stoic custom of considering only the conjunctions. To the traditional Chrysippian propositions he adds the disjunctive and the not-excluding alternative propositions. His *Introduction* is also one of the earliest sources for the doctrine on equivalence and conversion of compound propositions.

Although his interpretation of Stoic propositions is not strictly formal, Galen's presentation of the hypothetical syllogisms is in accord with the most rigid Stoic formalism. He clearly explains the unfamiliar Stoic terminology pertinent to syllogisms, and he accepts four of the five traditional indemonstrable formulae. These he supplements with the disjunctive and the not-excluding alternative syllogisms. He completes the treatment of the Stoic syllogism with his extraordinary doctrine on the conversion of hypothetical syllogisms.

Galen's teaching on the logic of propositions is a complete record of the state of Stoic logic in the second century. He not only summarizes the doctrine of the centuries immediately preceding but indicates the direction actually followed by subsequent logicians. His tendency to interpret Stoic Logic according to Peripatetic principles finds its culmination in classical logic. As this tendency so to interpret Stoic logic became more and more pronounced, the logic of the Stoa declined more and more until, having eventually lost its distinctiveness as a branch of dialectics, it was completely absorbed into Peripatetic logic. He proves himself beyond all doubt an independent thinker and not a mere compiler, for the logic of propositions comes from his hands colored by the touch of his originality. The study of this elementary logical treatise makes us strongly desire to know some of more extensive works of Galen, the great logician, who has given us *The Introduction to Dialectic*."

From: James W. Stakelum, *Galen and the Logic of Propositions*, Romae, Angelicum, 1940, pp. 90-91

ANCIENT THEORIES OF MEANING

"There were three ancient theories of meaning:

- (1) According to the Peripatetics, words mean thoughts, and thoughts stand for things.
- (2) According to the Epicureans, words directly mean things.
- (3) According to the Stoics, words mean sayables, (1) and sayables stand for things.

The Stoics agree with the Peripatetics and disagree with the Epicureans in maintaining that a semantic theory must be three-tiered. The Stoics disagree with the Peripatetics insofar as the intermediate items in their three-tiered theory are sayables and not thoughts.

Thus far, mere caricature: each of the theories I have sketched requires further elucidation; and each of the sketches would be regarded as wildly inaccurate by some scholars. I shall not attempt to replace the

caricatures by professional portraits; rather, I want to address one particular problem which the caricatures raise. If the Peripatetic and Stoic theories differ insofar as thoughts differ from sayables, then -- we may well wonder -- what exactly is the difference between sayables and thoughts, and how is Stoic saying related to Stoic thinking?

Several scholars, both ancient and modern, have denied that there is any substantive difference between the Peripatetic and the Stoic theories of meaning on the grounds that sayables are simply thoughts under a different name. Thus according to Simplicius, some people held that

the argument in the *Categories* is about thoughts (*peri noematon*); for Aristotle plainly says that it is about things which are said (*peri ton legomenon*), and things which are said, or sayables, are thoughts, as the Stoics too held. (*in Cat.* 10.2-4 = *FDS* 703) (2)

More recently it has been maintained that a sayable is "that which is merely an expressed thought"; for sayables "exist only insofar as they are thought and expressed in words. As ideas in the mind ... the *lekta* ... should be interpreted ... as something ... akin to the ideas of, for instance, classical British empiricism -- as a kind of mental images which precede and accompany our words and give them meaning" (3)

A weaker thesis has also found favour: sayables are not to be identified with thoughts, but they are logically dependent upon the activity of thinking. For "every species of *lekton* requires the utterance of some expressible object present to the mind. Does this entail that *lekta* only persist as long as the sentences which express them? ... there is no evidence to show that *lekta*, as distinct from the speaker and his reference, persist outside acts of thought and communication". (4)

These theses about sayables and thoughts are not mere conjectures. For there are several ancient texts which associate sayables with thoughts, and these texts have been taken to support either the strong view that sayables actually are thoughts or the weaker view that sayables are parasitic upon thoughts.

The issues are complicated, both from a philosophical and from an exegetical point of view. I shall first make a few abstract remarks; then look at the Peripatetic theory of meaning; and finally turn to the texts which associate thoughts with sayables." pp. 47-48

(1) I use the unlovely word "sayable" for the Greek *lekton*. I take it that *lekta* stand to saying as thoughts stand to thinking; but no decent English word stands to "say" as "thought" stands to "think".

(2) Note that Simplicius does not subscribe to this view of *lekta* (pace Long [1971] 80): he ascribes it to unnamed interpreters of Aristotle's *Categories*, and at e.g. *in Cat.* 397.10-12 he implicitly distinguishes *lekta* from *dianoemata*.

(3) Nuchelmans (1973) 52, 55.

(4) Long (1971) 97,98.

From: Jonathan Barnes, *Meaning, saying and thinking*. In *Dialektiker und Stoiker. Zur Logik der Stoa und ihrer Vorläufer*. Edited by Klaus Döring and Theodor Ebert, Stuttgart, Franz Steiner 1993 pp. 47-61

THE STOIC CLASSIFICATIONS OF ARGUMENTS

"In Sextus Empiricus' *Outlines of Pyrrhonism* (6) one finds the following Stoic definitions of the expressions 'premises', 'conclusion', and 'argument'

- (i) 'premises': the propositions assumed for the establishment of the conclusion,
- (ii) 'conclusion': the proposition which is established by the premises (7),
- (iii) 'argument': a whole composed of premises and a conclusion.

In terms of these definitions the questions I shall be attempting to answer are: for the Stoics what are the conditions under which the premises in an argument logically imply its conclusion? And, if the premises of an argument in fact imply its conclusion but not evidently so, how according to the Stoics may this relation of logical consequence be made evident? Before dealing with these questions, however, I present several classifications of Stoic arguments (see the outline of these classifications below).

The first division of the first classification of arguments is into valid and invalid arguments. An argument is valid "when the conditional having as its antecedent the conjunction formed from the premises of the argument and as its consequent the conclusion of the argument is true" (*P.H.* ii.137). An example of a valid argument is

(1) If it is day, it is light. It is day.

Therefore it is light.

Arguments which do not satisfy this condition are invalid.

Next valid arguments are divided into those which are true and those which are not true. A true valid argument is one of which both the conclusion and the premises are true (*P.H.* ii.138). An example of a true valid argument is (1) above when set forth during the day. Arguments which do not satisfy this condition are not true. An example of a not-true argument is the following when made during the day:

(2) If it is night, it is dark.

It is night.

Therefore it is dark.

Of true valid arguments some are demonstrative and some are not demonstrative. Demonstrative arguments are "those which conclude something non-evident through pre-evident premises".(8) An example of a demonstrative true valid argument, preserved by Sextus (*P.H.* ii.140), is

(3) If sweat flows through the surface of the skin, there exist imperceptible pores.

Sweat flows through the surface of the skin.

Therefore there exist imperceptible pores.

An argument not satisfying this condition is not demonstrative. Argument (1) is an example of an argument which is valid, true when set forth during the day, and not demonstrative. It will be shown subsequently that there was another kind of argument called undemonstrated, which provides an additional important category of arguments. It is not to be confused with a not-demonstrative argument.

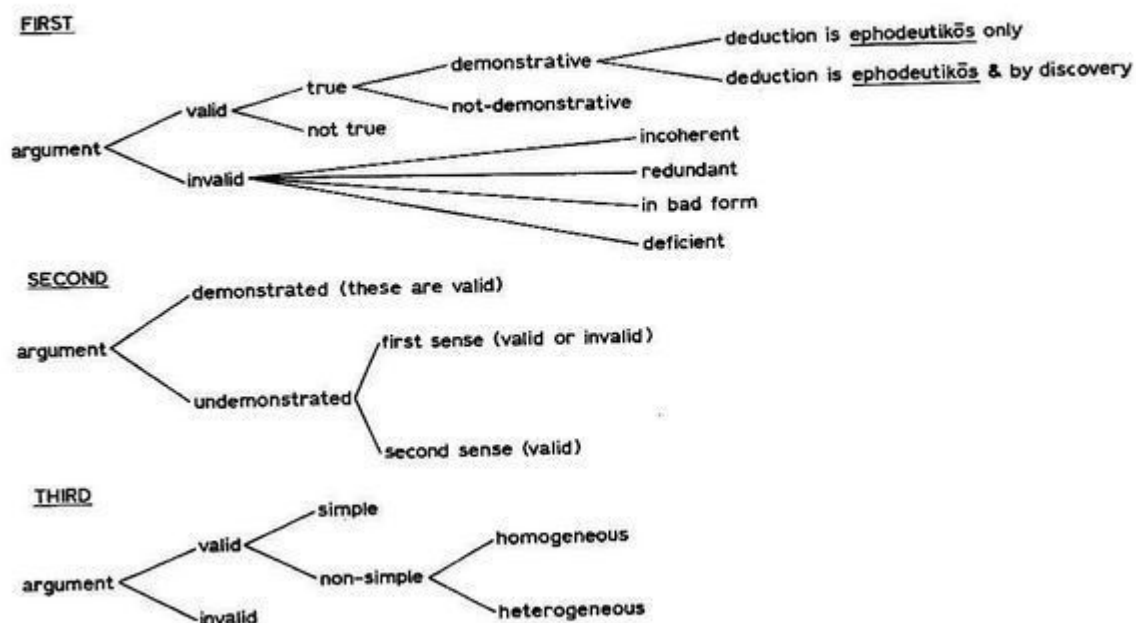
Of demonstrative true valid arguments "some lead us through the premises to the conclusion *ephodeutikos* only" (*P.H.* ii.141). I am not sure precisely what '*ephodeutikos*' means. Etymologically the word suggests 'advancing over a path towards something' and when the expression attaches to the word 'argument' a reasonable candidate for the 'something' would be the conclusion of the argument. But 'advancing over a path towards a conclusion' is a metaphorical description of arguments generally and it fails to bring out what is peculiar to the type of argument to which the label is here attached. I simply transliterate the expression. A kind of this type argument is said to be one which "depends upon belief and memory". One might well ask, 'What kind of argument doesn't?' An example of an argument which depends on belief and memory is

(4) If someone said to you that this man would be wealthy, this man will be wealthy.

This god said to you that this man would be wealthy. Therefore this man will be wealthy.

Sextus' comment on this argument is that we "assent to the conclusion not so much on account of the necessity of the premises as because we believe the assertion of the god" (*P.H.* ii.141-142).

SOME STOIC CLASSIFICATIONS OF ARGUMENTS



Contrasted with this type argument are those which "lead us to the conclusion not only *ephodeutikeis* but also by way of discovery" (*P.H.* ii.142). An example of such an argument is (3). The element of discovery

in this argument is the disclosure of the existence of pores through the fact that sweat flows through the surface of the skin. The element of belief in the argument, apparently sufficient to provide the *ephodeutikeis* component, is the "prior assumption that moisture cannot flow through a solid body" (*P.H.* ii.142).

The components of a 'demonstration' may be derived from one component of each division in this first classification, for a demonstration is a valid and true argument having a non-evident conclusion and disclosing that conclusion by the power of the premises (*P.H.* ii.143). I am uncertain as to the point of the last clause in Sextus' report. It appears to imply that the conclusion is obtained without the aid of assumptions external to the premises of the argument, although this would involve the existence of a class of demonstrative arguments different from those which are *ephodeutikeis*.

A second Stoic classification of arguments is also reported by Sextus, and it, too, ought to be kept in mind when thinking about deduction in Stoic logic. This classification begins from a division of arguments into demonstrated and undemonstrated. I take a demonstrated argument in this context to be one whose validity has been made evident. I say more subsequently about how the validity of arguments is made evident. An argument is undemonstrated in one of two senses. The first sense is the contradictory of that of 'demonstrated'. In this sense, then, an argument is undemonstrated if it has not been demonstrated (*Adv. Math.* viii.223), i.e., on my interpretation, if it has not been shown to be valid. In a second sense an argument is undemonstrated if it is immediately evident that it is valid (*ibid.*). This distinction may be brought out by noticing that the first sense is temporal inasmuch as an argument which is undemonstrated in that sense in 100 B.C. may be demonstrated in 50 B.C., while the second sense is non-temporal.⁽⁹⁾ An argument is undemonstrated in this second sense if it exhibits one of five forms of argument which are referred to respectively as the first undemonstrated, the second undemonstrated, etc. These forms are also called inference schemata, and I have more to say about them below. For now I merely give the forms with illustrative examples (Gould *The Philosophy of Chrysippus*, pp. 83-85):

The first undemonstrated

(5) If the first, the second.

The first.

Therefore the second.

If it is day, there is light.

It is day.

Therefore there is light.

The second undemonstrated

(6) If the first, the second.

Not the second.

Therefore not the first.

If it is day, there is light.

There is not light.

Therefore it is not day.

The third undemonstrated

(7) Not both the first and the second.

The first.

Therefore not the second.

Not both it is day and it is night.

It is day.

Therefore it is not night.

The fourth undemonstrated

(8) Either the first or the second.

The first.

Therefore not the second.

Either it is day or it is night.

It is day.

Therefore it is not night.

The fifth undemonstrated

(9) Either the first or the second.

Not the first.

Therefore the second.

Either it is day or it is night.

It is not day.

Therefore it is night.

A third classification divides valid arguments first into simple and non-simple (*Adv. Math.* viii.228). A simple valid argument is one having the form of one of the five undemonstrated argument forms. A non-simple valid argument is one 'woven together' out of simple valid arguments in order that it may be known to be 'valid' (*Adv. Math.* viii.229). There are two kinds of non-simple arguments, one formed from two or more simple arguments all of the same form, and the other composed from two or more simple arguments not of the same form. The former is a homogeneous non-simple and the latter, a heterogeneous non-simple argument (*ibid.*). An example of a homogeneous non-simple argument is

(10) If it is day, then if it is day it is light.

It is day.

Therefore it is light.

For upon analysis it may be seen to have been compounded from two simple arguments having the form of the first undemonstrated. Analysis of this argument is carried out in accordance with the following 'dialectical theorem':

(11) Whenever we have premises from which a certain conclusion can be validly deduced, potentially we have also that conclusion among the premises, even if it is not stated explicitly.(10)

One analyzes (10) by drawing the conclusion from the first two premises in accordance with the first undemonstrated inference schema, thus getting

(12) If it is day, then if it is day it is light.

It is day.

Therefore if it is day, it is light.

Then by the theorem stated in (11) one gets as premises

(13) If it is day, then if it is day it is light.

It is day.

If it is day, it is light.

And by another application of the first inference schema one gets the conclusion in (10)." pp. 152-157

(6) ii.135-136. This work will be referred to in the remainder of the paper as *P.H.*

(7) Thomas has rightly pointed out that the intent here must have been something like "the proposition which is allegedly established by the premises". Otherwise every conclusion would be the conclusion of a valid argument.

(8) *P.H.* ii.140. Sextus reports (*P.H.* ii.97-98) that the 'dogmatists' distinguished three kinds of non-evident objects. Some are absolutely non-evident; these are those which are not of the sort to fall under our apprehension, e.g., that the stars are even in number. Some are on occasion non-evident; these are of a sort to be evident but are made non-evident on occasion by external circumstances, e.g., as a city in which I am not present now is to me. Finally, some are naturally non-evident; these are naturally incapable of falling under our clear apprehension, e.g., that there are imperceptible pores.

(9) I am indebted to John Corcoran for having suggested to me this feature of the distinction.

(10) *Adv. Math.* viii.231. I discuss this theorem below (p. 18) in conjunction with other Stoic rules of inference.

From: Joseph Gould, Deduction in Stoic Logic, in: *Ancient logic and its modern interpretations. Proceedings of the Buffalo Symposium on Modernist Interpretations of Ancient Logic, 21 and 22 April, 1972.* Edited by Corcoran John. Dordrecht: Reidel 1974. pp. 151-168

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